Department of Mathematics

Dinakrushna College, Jaleswar

Core I, Ist Semester

Paper : Calculus-I

Section–A

1. a) $\frac{d}{dx}$ (sin hx) = _____

b) $\frac{d}{dx}$ (tan hx) = _____

c) Find the approximate value of cosh (–2)

d) Find the exact neumerical value of sin h (–3 ln2).

e) Find $\frac{dy}{dx}$ if y = ln (coshx).

f) Find $\frac{dy}{dx}$ of $y = \tanh^{-1}\left(\frac{x^2-1}{x^2+1}\right)$

g) Evaluate $\int$ Cosech^2 (3x) dx

h) $\int_0^{\ln 3} \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$ = ?

i) If y = ln (ax+b) then y_n = ?

j) If y = sin (ax+b) then y_n = ?

k) What is ‘Monotone function Theorem’?

l) Write the 2nd derivative Test for determining intervals of concavity?

m) Using Ist derivative test, classify the given critical numbers as relative minimum, relative maximum or neither

a) $f(x) = \frac{e^{-x^2}}{3-2x}$ at x =1, $\frac{1}{2}$

b) f(x) = x–2 sinx, x $\in$ [0, 2π]

n) Determine the intervals of increase and decrease for the following functions.

a) $f(x) = \frac{x}{x^2+1}$, (b) $f(x) = x^2e^{-3x}$.

P.T.O.

o) Evaluate i) $\lim_{x\to\infty} \frac{2x^3-4x+7}{4x^3+2x^2+8}$ (ii) $\lim_{x\to\infty} \frac{(2x+5)(x-3)}{(7x-2)(4x+1)}$

p) Evaluate $\lim_{x\to\infty} e^x \sin x$.

q) Find the vertial asymptote of the function f(x) = $\frac{3x+5}{7-x}$

r) Find the horizontal asymptote of the graph of the function f(x) = $\frac{x^3+1}{x^3-8}$?

s) Find the asymptotes parallel to either axes : $x^2y^2=9(x^2+y^2)$

t) Find asymptotes to the curves : x = $\frac{t^2}{1+t^3}$, y = $\frac{t^2+2}{1+t}$

u) If $\lim_{x\to 0} \frac{e^{ax}-e^{-ax}}{\ln(1+bx)}$ = 1 with b – a = 3, find a and b?

v) Evaluate $\lim_{x\to 0} \frac{e^{3x}-3x-1}{1-\cos x}$

w) Find $\lim_{x\to 0+} \frac{x \sin\frac{1}{x}}{\sin x}$ if it exists.

x) Write reduction formula for $\int \cot^n x\, dx$

y) Write Walli's sine formulae and Cosine formula.

z) Write the formula to find the volume of solid by washer method?

2. a) Write the formulae to find the volume of the solid by cylindrical shell method?

b) Eliminate the parameter in the following

i) x = 4t–1, y = 3t+2

ii) x = 3+2cost, y = 2+4 sint, t$\in$ [0, 2π].

iii) x = $\frac{2t^3}{1+t^2}$, y = $\frac{2t^2}{1+t^2}$

iv) x = 5 cost, y = 2 sint, t $\in$[0, 2π]

c) Write the parametric equation of Astroid : $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

d) Write the equation of the following:

i) Semicubical parabola

ii) Cycloid

iii) Epicycloid

iv) Hypocloid

v) Trochoid

e) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the given point without eliminating the parameter :

i) $x=\sqrt{t}$, $y = 2t+7$, $t=1$,

ii) $x = \theta + \cos\theta$, $y = 1+\sin\theta$, $\theta = \frac{\pi}{4}$

f) Find the points of the intersection of the following lines with co-ordinate planes :

i) $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z-6}{3}$

ii) $x = 6+3t$, $y = 2-t$, $z = 2t$

g) Find the length of the circumference of the circle $x^2+y^2 = 25$?

h) Write the formula to find the arc length for parametric curves?

i) Write the formula to find the surface area of revolution about x–axis?

j) Write the formula to find the surface area of revolution about y–axis?

k) Find the area of the surface generated by revolving the given curve about y–axis?

i) $x = \sqrt{9-y^2}$, $-2 \le y \le 2$

ii) $x = 9y+1$, $0 \le y \le 2$

l) Find he asymptotes of the hyperbola $\frac{(y+4)^2}{3} - \frac{(x-2)^2}{5} = 1$

m) Write the centre and radius of the sphere, $x^2+y^2+z^2+2ux+2vy+2wz+d = 0$?

n) If $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$ then $\lim_{t\to t_0} \vec{r}(t) = ?$

o) Find the limit

i) $\lim_{t\to\infty} \left\{ \frac{t^3+1}{4t^3+2}\hat{i} + \frac{\hat{1}}{t}\hat{j} \right\}$ (ii) $\lim_{t\to 1} \left\{ \frac{2}{t^2}\hat{i} + \frac{\ln t}{t^2-1}\hat{j} + \cos 3t\,\hat{k} \right\}$

P.T.O.

p) Write the condition for a vector valued function r(t) is continous at $t = t_0$?

q) Differentiate the following scalar function. $\{ x\hat{i} + (x+1)\hat{j} \}$ $\{(2x)\hat{i} - (3x)^2\hat{j} \}$

r) Evaluate $\int (e^t, e^{-t}, 3t^2)\, dt$

Section – B

1)a) Find the exact numerical value of

i) Cos h (–ln2)

ii) sin h (–3 ln2)

b) Approximate the following expression to four decimal place.

i) cos h(–2)

ii) sin h (ln3).

c) Find the derivative of the functions with respect to x.

i) sin h $(2x^2 - 6x + 4)$

ii) $e^{\cot bx}$

iii) $\sqrt{4x + \cosh^2(5x)}$

d) Prove that $\sinh 3x = 4 \sinh^3 x + 3 \sin hx$.

e) Find $\frac{dy}{dx}$ of $y = \tan^{-1}(\tanh \frac{x}{2})$

f) Evaluate $\lim_{x \to \infty} (\text{Cosh}^{-1} x - \ln x)$

g) Find $\frac{dy}{dx}$ of $y = \sin h^{-1} \sqrt{x} + \cosh^{-1}(2x)$

h) Evaluate the following integrals.

i) $\int \text{Cosech}^2 (3x)dx$

ii) $\int_0^{\ln 3} \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$

i) Evaluate $\int \sinh^6 x \cosh x \, dx$

j) Evaluate $\int \frac{1}{\sinh x + 2\cosh x}\, dx$

k) Show that $\int \frac{dx}{\sqrt{x^2 + \sqrt{256}}} = \sinh^{-1} \frac{x}{4} + x$

l) Find y_4 of $y = x^4 \cos x$.

m) Find y'' for $x^2+x^4 = 10$

n) Find the point of inflextion for the function

$f(x) = x^4 + 4x^3 - 18x^2 + 9x - 3$.

o) Examine each of the following functions for concavity, convexity and points of inflexion

i) $f(x) = x^3$ (ii) $f(x) = \frac{x-2}{x-3}$

p) Determine the intervals of increase and decrease fot the following functions.

i) $f(x) = \frac{x}{x^2+1}$ (ii) $f(x) = x^2e^{3x}$ (iii) $f(x) = (\ln x)^2$

q) Determine whether the graph of the function $f(x) = x^{\frac{1}{3}}(x-4)$ have a vertical tangent or a cusp.

r) Sketch the graph $f(x) = \frac{x}{1-x}$

s) Evaluate $\lim_{x\to 0} \frac{e^{3^x} - 3x - 1}{1-\cos x}$

t) Evaluate $\lim_{x\to 0} \frac{x^2 + \sin x^2}{x^2 + x^3}$

u) Evaluate $\lim_{x\to\infty} \frac{x(\pi + \sin x)}{x^2+1}$

v) Evaluate $\lim_{x\to\frac{\pi}{2}} (1-\sin x)\tan x$

w) Evaluate $\lim_{x\to 1} x^{\frac{1}{1-x}}$

x) Evaluate $\int \cos^5 x\, dx$ using reduction formula.

y) Evaluate $\int \sec^7 x\, dx$

z) Evaluate $\int_0^{\frac{\pi}{2}} \sin^{10} x\, dx$, using Walli's formula.

2. a) Evaluate $\int \sin^4 x \cdot \cos^6 x\, dx$

b) Find the volume of the solid generated when the area bounded by the curve $y = 3-2x$, $y = 2$ and $x = 0$ is revolved about y-axis?

c) Using Washer method, determine the volume of the solid obtained when the region enclosed by the curves $x = \sqrt{y}$ and $x = \frac{y}{4}$ is revolved above x–axis.

P.T.O.

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t) Evaluate $\lim_{x\to 0} \frac{x^2 + \sin x^2}{x^2 + x^3}$

u) Evaluate $\lim_{x\to\infty} \frac{x(\pi + \sin x)}{x^2+1}$

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P.T.O.

w) Find the equations of the normal to the ellipsoid $x^2+2y^2+2z^2 = 5$ at (1,1,1)

x) Let $\vec{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$. Find $\lim_{t\to 2} \vec{r}(t)$. $(\vec{r}'(t) \times \vec{r}''(t))$

y) The aceleration of a particle at any time t is $\frac{d^2\vec{R}}{dt^2} = e^{2t}\hat{i} + e^{-t}\hat{j} + 2t\hat{k}$. Find velocity $\vec{v}(t)$ given that $\vec{v} = \hat{i}+\hat{j}$ when t = 0.

Section – C

1)a) Show that $\int \sinh^4 x\, dx = \frac{1}{32}\sin h4x - \frac{1}{4}\sinh 2x + \frac{3}{8}x + c$.

b) If $y = \cosh(\sin^{-1}x)$, show that $(1-x^2)\, y_{n+2} - (2n+1)xy_{n+1} - (x^2+1)\, y_n = 0$.

c) If $y = (\sin^{-1}x)^2$, Prove that $(1-x^2)\, y_{n+1} - (2n+1)xy_{n+1} - n^2\, y_n = 0$.

d) Determine a, b and c such that the graph of $f(x) = ax^3+bx^2+c$ has an infection point and slope 1 at (–1,2).

e) Find Constants A, B, and C, such that the function $f(x) = Ax^3+Bx^2+C$ will have relative extremum at (2,11) and an inflextion point at(1,5)?

f) Trace the curve $(x^2-1)(y^2-4) = 4$.

g) Trace the curves $x = a(t+\sin t)$, $y = a(1+\cos t)$, $-\pi \le t \le \pi$

h) Trace the curve $r^2 = a^2 \sin 2\theta$,

i) Find all horizontal asymptotes to the graph of the function $f(x) = \left(\frac{x+3}{x+2}\right)^{2x}$ Using L′ Hospital's rule.

j) If $C(x) = \frac{x^2}{25} - 3x + 100$, find the minimum value of cost?

k) Find the volume of the solid generated by revolving around the y-axis, the area between the curves y=x, x=0 and $y = \sqrt{x+2}$.

l) Determine whether the following lines intersect, are parallel or are skew.

$$\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{4} \quad \text{and} \quad \frac{x+2}{4} = \frac{y}{-3} = \frac{z+1}{1}$$

P.T.O.

m) Compute the length of cure $x = t \sin t$, $y = t\cos t$, from $t = 0$ to $t = 2\pi$.

n) Find the surface of the formed by the revolution about x–axis of the loop of the curve $x=t^2$, $y = t - \frac{t^3}{3}$.

o) Sketch the graph of hyperbola $\frac{(y-3)^2}{9} - \frac{(x-4)^2}{25} = 1$

p) Find the equation of the right circular cylinder whose guiding curve is $x^2+y^2+z^2 = 9$, $x-y+z = 0$.

q) Find the equations of λ and μ generators of the hyperboliod of one sheet $\frac{x^2}{4} + y^2 - z^2 = 49$ which passes through the point (10,5,1).

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